(i) $\sin(\sin^{-1} x) = x$ for all $x \in [-1, 1]$

(ii) $\cos(\cos^{-1} x) = x,$ for all $x \in [-1, 1]$

(iii) $\tan(\tan^{-1} x) = x$ for all $x \in R$

(iv) $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x,$ for all $x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec(\sec^{-1} x) = x,$ for all $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot(\cot^{-1} x) = x,$ for all $x \in R.$

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$$\sin^{-1}(\sin\theta) = \begin{cases} -\pi-\theta & \text{, if } \theta \in [-3\pi/2, -\pi/2] \\ \theta & \text{, if } \theta \in [-\pi/2, \pi/2] \\ \pi-\theta & \text{, if } \theta \in [\pi/2, 3\pi/2] \\ -2\pi+\theta & \text{, if } \theta \in [3\pi/2, 5\pi/2] \end{cases} \quad \text{and so on.}$$

Similarly,

$$\cos^{-1}(\cos\theta) = \begin{cases} -\theta & \text{, if } \theta \in [-\pi, 0] \\ \theta & \text{, if } \theta \in [0, \pi] \\ 2\pi-\theta & \text{, if } \theta \in [\pi, 2\pi] \\ -2\pi+\theta & \text{, if } \theta \in [2\pi, 3\pi] \end{cases} \quad \text{and so on.}$$

$$\tan^{-1}(\tan\theta) = \begin{cases} -\pi-\theta & \text{, if } \theta \in [-3\pi/2, -\pi/2] \\ \theta & \text{, if } \theta \in [-\pi/2, \pi/2] \\ \theta-\pi & \text{, if } \theta \in [\pi/2, 3\pi/2] \\ \theta-2\pi & \text{, if } \theta \in [3\pi/2, 5\pi/2] \end{cases} \quad \text{and so on.}$$

PROPERTY III   (i) $\sin^{-1}(-x) = -\sin^{-1}(x)$,   for all $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$,   for all $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$,   for all $x \in R$

(iv) $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}x$,   for all $x \in (-\infty, -1] \cup [1, \infty]$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$,   for all $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x$,   for all $x \in R$

IV. 1. $\sin^{-1}x = \operatorname{cosec}^{-1}\left(\dfrac{1}{x}\right), -1 \leq x \leq 1$

$\operatorname{cosec}^{-1}x = \sin^{-1}\left(\dfrac{1}{x}\right), x \leq -1 \text{ or } x \geq 1$

2. $\cos^{-1}x = \sec^{-1}\left(\dfrac{1}{x}\right), -1 \leq x \leq 1$

$\sec^{-1}x = \cos^{-1}\left(\dfrac{1}{x}\right), x \leq -1 \text{ or } x \geq 1$

3. $\tan^{-1}x = \cot^{-1}\left(\dfrac{1}{x}\right), x > 0$

$\tan^{-1}x = \cot^{-1}\left(\dfrac{1}{x}\right) - \pi, x < 0$

$\cot^{-1}x = \tan^{-1}\left(\dfrac{1}{x}\right), x > 0$

$\cot^{-1}x = \pi + \tan^{-1}\left(\dfrac{1}{x}\right), x < 0$

III. 1. $\quad \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$ for $-1 \leq x \leq 1$

2. $\quad \tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}$ for $x \in R$

3. $\quad \sec^{-1}x + \operatorname{cosec}^{-1}x = \dfrac{\pi}{2}$ for $x \leq -1$ or, $x \geq 1$

6.

$$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } xy<1 \\[3mm] \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } x>0,\ y>0 \text{ and } xy>1 \\[3mm] -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } x<0,\ y<0 \text{ and } xy>1 \end{cases}$$

$$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } xy>-1 \\[3mm] \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } x>0,\ y<0 \text{ and } xy<-1 \\[3mm] -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } x<0,\ y>0 \text{ and } xy<-1 \end{cases}$$

7.

$$\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \begin{array}{l}\text{if } -1\le x,\ y\le 1 \text{ and } x^2+y^2\le 1 \\ \quad\quad\quad\text{or} \\ \text{if } xy<0 \text{ and } x^2+y^2>1\end{array} \\[6mm] \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \text{if } 0<x,\ y\le 1 \text{ and } x^2+y^2>1 \\[4mm] -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \text{if } -1\le x,\ y<0 \text{ and } x^2+y^2>1 \end{cases}$$

$$\sin^{-1}x - \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \begin{array}{l}\text{if } -1\le x,\ y\le 1 \text{ and } x^2+y^2\le 1 \\ \quad\quad\quad\text{or} \\ \text{if } xy>0 \text{ and } x^2+y^2>1\end{array} \\[6mm] \pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \text{if } 0<x\le 1,\ -1\le y\le 0 \text{ and } x^2+y^2>1 \\[4mm] -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \text{if } -1\le x<0,\ 0<y\le 1 \text{ and } x^2+y^2>1 \end{cases}$$

8.

$$\cos^{-1}x + \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le x,\ y\le 1 \text{ and } x+y\ge 0 \\[3mm] 2\pi - \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le x,\ y\le 1 \text{ and } x+y\le 0 \end{cases}$$

$$\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le x,\ y\le 1 \text{ and } x\le y \\[3mm] -\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le y\le 0,\ 0<x\le 1 \text{ and } x\ge y \end{cases}$$

9.

$$2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}) & , \text{ if } -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\[2mm] \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{ if } \dfrac{1}{\sqrt{2}} \le x \le 1 \\[2mm] -\pi - \sin(2x\sqrt{1-x^2}) & , \text{ if } -1 \le x \le -\dfrac{1}{\sqrt{2}} \end{cases}$$

$$3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3) & , \text{ if } -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\[2mm] \pi - \sin^{-1}(3x-4x^3) & , \text{ if } \dfrac{1}{2} < x \le 1 \\[2mm] -\pi - \sin^{-1}(3x-4x^3), & \text{ if } -1 \le x < -\dfrac{1}{2} \end{cases}$$

10.

$$2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2-1) & , \text{ if } 0 \le x \le 1 \\[2mm] 2\pi - \cos^{-1}(2x^2-1), & \text{ if } -1 \le x \le 0 \end{cases}$$

$$3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3-3x) & , \text{ if } \dfrac{1}{2} \le x \le 1 \\[2mm] 2\pi - \cos^{-1}(4x^3-3x), & \text{ if } -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\[2mm] 2\pi + \cos^{-1}(4x^3-3x), & \text{ if } -1 \le x \le -\dfrac{1}{2} \end{cases}$$

11.

$$2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & , \text{ if } -1 < x < 1 \\[3mm] \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & , \text{ if } x > 1 \\[3mm] -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \text{ if } x < -1 \end{cases}$$

$$3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) & , \text{ if } -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}} \\[3mm] \pi + \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) & , \text{ if } x > \dfrac{1}{\sqrt{3}} \\[3mm] -\pi + \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right), & \text{ if } x < -\dfrac{1}{\sqrt{3}} \end{cases}$$

PROPERTY XII

(i) $2 \tan^{-1} x = \begin{cases} \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & , \text{if} -1 \le x \le 1 \\[4mm] \pi - \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & , \text{if } x > 1 \\[4mm] -\pi - \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & , \text{if } x < -1 \end{cases}$

(ii) $2 \tan^{-1} x = \begin{cases} \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) & , \text{if } 0 \le x < \infty \\[4mm] -\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) & , \text{if} -\infty < x \le 0 \end{cases}$